

$SU(3)$  SKYRMIONS <sup>1</sup>

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## ABSTRACT

The consideration of the bound skyrmions with large strangeness content is continued. The connection between  $B = 2$   $SO(3)$ -hedgehog and  $SU(2)$ -torus is investigated and the quantization of the dipole-type configuration with large strangeness content is described.

1. At the conference Quarks-94 we presented preliminary results of our studies of  $SU(3)$  skyrmions - configurations of chiral fields which are described by 8 functions of 3 variables [1]. Using a special program allowing for the minimization of the energy functionals depending on ten functions (with 2 unitarity conditions imposed) we obtained the following results.

In the sector with baryon number  $B = 1$  the  $SU(2)$  hedgehog and in the sector with  $B = 2$  the  $SU(2)$  torus are local minima in  $SU(3)$  configuration space. The local rotations in "strange" direction do not allow to decrease the energy of mentioned static configurations which are believed to be absolute minima of static energy for  $B = 1$  and 2.

The new local minimum was found in the  $B = 2$  sector with large scalar strangeness content  $SC$ , close to 0.5. This configuration was obtained starting with two  $B = 1$  skyrmions located in  $(u, s)$  and  $(d, s)$   $SU(2)$  subgroups of  $SU(3)$  in optimal (attractive) relative orientation and optimal distance between topological centers. Applying our minimization algorithm we obtained the configuration of the dipole type consisting of two deformed  $B = 1$  hedgehogs in different  $SU(2)$  subgroups with binding energy about half of that of the torus, i.e. about 0.04 of the static energy of  $B = 1$  hedgehog.

We investigated the flavor symmetric ( $FS$ ) case when all meson masses in the effective lagrangian are equal to the pion mass. The configuration found in [1] possesses remarkable symmetry properties.

Later we extended our studies to flavor symmetry broken ( $FSB$ ) case with kaon mass included into the lagrangian. Here we shall discuss our latest results concerning  $FSB$  effects, the connection between  $SO(3)$  hedgehog and  $SU(2)$  torus, and the quantization of  $SU(3)$  skyrmions.

2. The parametrization we used for the unitary  $SU(3)$  matrix  $U$  incorporating

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8 chiral fields is

$$U = U_L(u, s)U(u, d)U_R(d, s) \quad (1)$$

where one of  $SU(2)$ -matrices, e.g.  $U(u, d)$  depends on two parameters:

$$U(u, d) = \exp(ia\lambda_2) \exp(ib\lambda_3) \quad (2)$$

The chiral and flavor symmetry breaking terms in the lagrangian density can be presented in the following form:

$$M_{m.t.} = \frac{1}{8}F_\pi^2 m_\pi^2 (2 - v_1 - v_2) + \frac{1}{8}(2F_K^2 m_K^2 - F_\pi^2 m_\pi^2)(1 - v_3) \quad (3)$$

The real parts of diagonal matrix elements of matrix  $U$  for this case are expressed through the functions parametrizing our ansatz according to:

$$\begin{aligned} v_1 &= c_b c_a f_0 - s_b c_a f_3 \\ v_2 &= c_b c_a q_0 + s_b c_a q_3 \\ v_3 &= f_0 q_0 - f_3 q_3 + s_a [s_b (f_1 q_2 - f_2 q_1) - c_b (f_2 q_2 + f_1 q_1)] \end{aligned} \quad (4)$$

The functions  $f_i$  and  $q_i$  parametrize the  $(u, s)$  and  $(d, s)$  solitons in the following way:

$$\begin{aligned} \tilde{U}_L(u, s) &= f_0 + i(f_1 \tau_1 + f_2 \tau_2 + f_3 \tau_3) \\ \tilde{U}_R(d, s) &= q_0 + i(q_1 \sigma_1 + q_2 \sigma_2 + q_3 \sigma_3) \end{aligned} \quad (5)$$

where  $\tau_k$  and  $\sigma_k$  are the Pauli matrices of  $(u, s)$  and  $(d, s)$   $SU(2)$  subgroups of  $SU(3)$ . Scalar strangeness content of the configuration is defined as

$$C_S = \frac{1 - \bar{v}_3}{3 - \bar{v}_1 - \bar{v}_2 - \bar{v}_3}$$

with averaging/integration over the whole soliton being performed.

Special care was taken to improve the baryon number stabilization of the configuration, which was especially important when  $FSB$  terms were added to the lagrangian. The inclusion of the kaon mass term leads to the increase of masses of solitons up to  $1982 Mev$  and  $3895 Mev$  for  $B = 1$  hedgehog and  $B = 2$  dipole - instead of  $1702 Mev$  and  $3334 Mev$  correspondingly in flavor symmetric case. Here we take  $F_K = F_\pi = 186 Mev$  and  $e = 4.12$ , as in [1]b. The uncertainty of the calculation does not exceed  $\sim 5$  and  $\sim 10 Mev$  for  $B = 1$  and  $2$ .

The other effect of  $FSB$  is the reduction of the dimensions of the configuration: the kaon cloud which the strange skyrmion consists of is smaller in dimensions by a factor about  $\sim 1.5$  in comparison with the pion cloud in nonstrange  $FS$  case. In the Fig.1 we show the equal mass density lines for the configurations of lowest energy for  $FSB$  case.

As it was stressed already the configuration we found possesses remarkable symmetry properties: the functions  $a$  and  $b$  have symmetry corresponding to the azimuthal winding  $n = 2$ . In the Fig.2 the function  $a$  is shown in the plane  $z = 0$  for

$d_0 = 0.75Fm$  and for  $d_0 = 0.02$ , also for  $FSB$  case. The difference in topological properties of both configurations is quite clear: the configuration of more complicated structure has higher energy.

The physics consequences of the existence of dipole-type configuration will be considered in Section 4.

3. We investigated the connection between  $SO(3)$  hedgehog and  $SU(2)$  torus, both  $B = 2$  configurations. Previously we calculated the masses of both configurations as functions of chiral symmetry breaking parameter  $\mu^2$  [2],[3]. For  $\mu = 0$  the torus is lower in energy by  $0.2\frac{F_\pi}{e}$ , with increasing  $\mu$  the difference in masses decreases and changes sign at  $\mu^2 = 0.04Gev^2$ . For realistic values of chiral symmetry breaking parameter,  $\mu^2 = 8.44m_\pi^2$  the mass of  $SO(3)$  hedgehog is smaller than the mass of the torus by about  $\sim 30 - 40Mev$  - depending on the parameters of the model.

A special ansatz allowing for connection between  $SO(3)$  hedgehog and the  $(u, d)$   $SU(2)$  torus was proposed by B.Schwesinger. It is the following one:

$$U = U_L U_4 U_M U_8 U_4^\dagger U_L^\dagger \quad (6)$$

$$\begin{aligned} U_L &= \exp(-i\phi\lambda_2)\exp(2ig_1\lambda_3) \\ U_M &= \exp(-ig_1\lambda_3)\exp(i\chi)\exp(ig_1\lambda_3) \\ U_4 &= \exp(ig_2\lambda_4) \\ U_8 &= \exp(-i\rho\lambda_8/\sqrt{3}) \\ g_1 &= \frac{1}{2}\arccos(c_\theta/c_{g_2}) \\ g_2 &= \arccos(\sqrt{c_\theta^2 + s_\theta^2 s_\gamma^2}) \end{aligned} \quad (7)$$

Here  $\theta = \arccos(z/r)$  is the polar angle,  $\phi = \arctg(y/x)$  is the azimuthal angle in usual coordinates  $(x, y, z)$  space.  $c_\theta = \cos(\theta)$ ,  $c_\gamma = \cos(\gamma)$ . The parameter  $\gamma$  changes in the interval  $(0, \pi/2)$ .

The 8  $SU(3)$  rotated Cartan-Maurer currents defined as

$$U_8 U_4^\dagger U_L^\dagger U^\dagger d_i U U_L U_4 U_8^\dagger = iL_{k,i}\lambda_k \quad (8)$$

in this case are equal to

$$\begin{aligned} L_{1,i} &= 2c_a[-l_{1,i}(q_2^2 + q_3^2) + l_{2,i}(q_1q_2 - q_0q_3)] + (1 + c_a^2)l_{3,i}(q_0q_2 + q_1q_3) + m_{1,i} \\ L_{2,i} &= 2c_a[l_{1,i}(q_0q_3 + q_1q_2) - l_{2,i}(q_1^2 + q_3^2)] + (1 + c_a^2)l_{3,i}(q_2q_3 - q_0q_1) + m_{2,i} \\ L_{3,i} &= 2c_a[l_{1,i}(q_1q_3 - q_0q_2) + l_{2,i}(q_0q_1 + q_2q_3)] - (1 + c_a^2)l_{3,i}(q_1^2 + q_2^2) + m_{3,i} \\ L_{4,i} &= -s_a[l_{1,i}q_1 + l_{2,i}q_2 + c_al_{3,i}(q_3 - s_b)] - d_ia(q_0 - c_b) \\ L_{5,i} &= s_a[-l_{1,i}q_2 + l_{2,i}q_1 + c_al_{3,i}(q_0 - c_b)] - d_ia(q_3 - s_b) \\ L_{6,i} &= s_a[l_{1,i}(q_3 + s_b) + l_{2,i}(q_0 - c_b) - c_al_{3,i}q_1 - d_iaq_2] \end{aligned}$$

$$L_{7,i} = s_a[l_{1,i}(q_0 - c_b) - l_{2,i}(q_3 + s_b) + c_a l_{3,i}q_2 - d_i a q_1]$$

$$L_{8,i} = -d_i b / \sqrt{3} \quad (9)$$

Here functions  $q_i$  parametrize the  $SU(2)$  matrix  $U_M$  embedded into  $SU(3)$ , similar to expressions (5) above and should be expressed through functions  $g_1$  and  $\chi$  according to (7).  $U(L)$  is also  $(u, d)$   $SU(2)$  matrix embedded into  $SU(3)$ .  $l_{k,i}$  and  $m_{k,i}$  are  $SU(2)$  Cartan-Maurer currents connected with  $U_L$  and  $U_M$ ,  $k, i = 1, 2, 3$ . The expressions (9) should be substituted into the expression for the static energy of solitons [1], see also (11) below.

This (local phase) parametrization allows to connect the  $SO(3)$  hedgehog and  $SU(2)$ -torus: for  $\gamma = 0$  strangeness changing angle equals to polar angle,  $g_2 = \theta$ , and with special choice of profiles  $\rho$  and  $\chi$  we obtain the  $SO(3)$  hedgehog. Further minimization using our algorithm did not lead to any decrease of the energy and to changes of the configurations. Therefore, we conclude that the  $SO(3)$  hedgehog with  $B = 2$  is a local minimum in  $SU(3)$  configuration space.

For  $\gamma = \pi/2$  we started with energy which is considerably greater than energy of the torus. On the start  $g_2 = 0$ , but strangeness content of the configuration is not zero since the function  $\rho$  is different from zero. The function  $g_2$  remains to be 0 during minimization, and  $\rho$  decreases and tends to 0. SC also tends to 0. The minimization performed on the work-station at Siegen University during several months allowed to get the torus-like configuration corresponding to the local minimum in  $(u, d)$   $SU(2)$  subgroup of  $SU(3)$ .

When we started with some intermediate values of  $\gamma$  we received configurations with strong enhancement in the mass density and baryon number density along  $z$ -axis. However, big losses of the  $B$ -number during minimization did not allow us to come to some firmly established final configurations. Most probably, these final configurations are above both  $SO(3)$  hedgehog and  $SU(2)$  torus. The methods we used certainly should be improved to investigate the complicated ansatz of the type (6), (7). The final mesh effects are essential for the ansatz of this type, and special care should be taken to remove these effects.

4. The quantization of zero modes of  $SU(3)$  skyrmions was investigated by one of the authors [4], and here we discuss it briefly for completeness. The quantization of skyrmions was made previously in the following cases: for  $SU(2)$  skyrmions rotated in the  $SU(3)$  collective coordinates space [5, 6], and also for  $SO(3)$  skyrmions [7]. These two cases do not exhaust the possible variety of  $SU(3)$  skyrmions, and, really, the dipole type configuration with strangeness content close to 0.5 [1] cannot be quantized using the known algorithm. The generalization of this quantization procedure for arbitrary  $SU(3)$  skyrmions seems to be actual.

Consider first the rotation energy of solitons. The kinetic term is:

$$E_{rot}^{kin} = \frac{F_\pi^2}{16} (\tilde{\omega}_1^2 + \dots + \tilde{\omega}_8^2) \quad (10)$$

The Skyrme term makes the contribution

$$\begin{aligned}
E_{rot}^{Sk} = & \frac{1}{8e^2} \left\{ \vec{s}_{12}^2 + \vec{s}_{23}^2 + \vec{s}_{31}^2 + \vec{s}_{45}^2 + \vec{s}_{67}^2 + \frac{3}{4} \left( \vec{s}_{48}^2 + \vec{s}_{58}^2 + \vec{s}_{68}^2 + \vec{s}_{78}^2 \right) + \right. \\
& + \frac{1}{4} \left( \vec{s}_{46}^2 + \vec{s}_{47}^2 + \vec{s}_{56}^2 + \vec{s}_{57}^2 + \vec{s}_{14}^2 + \vec{s}_{15}^2 + \vec{s}_{16}^2 + \vec{s}_{17}^2 + \vec{s}_{24}^2 + \vec{s}_{25}^2 + \vec{s}_{26}^2 + \vec{s}_{27}^2 + \vec{s}_{34}^2 + \vec{s}_{35}^2 + \vec{s}_{36}^2 + \vec{s}_{37}^2 \right) + \\
& + \frac{\sqrt{3}}{2} \left( \vec{s}_{84}(\vec{s}_{16} + \vec{s}_{34} - \vec{s}_{27}) + \vec{s}_{85}(\vec{s}_{17} + \vec{s}_{26} + \vec{s}_{35}) + \vec{s}_{86}(\vec{s}_{14} + \vec{s}_{25} - \vec{s}_{36}) + \vec{s}_{87}(\vec{s}_{15} - \vec{s}_{24} - \vec{s}_{37}) \right) + \\
& \left. + \frac{3}{2} \left( \vec{s}_{12}(\vec{s}_{45} + \vec{s}_{76}) + \vec{s}_{23}(\vec{s}_{47} + \vec{s}_{65}) + \vec{s}_{13}(\vec{s}_{64} + \vec{s}_{75}) + \vec{s}_{45}\vec{s}_{67} \right) \right\} \quad (11)
\end{aligned}$$

with  $\vec{s}_{ik} = \tilde{\omega}_i \vec{L}_l - \tilde{\omega}_k \vec{L}_i$ ,  $i, k = 1, 2, \dots, 8$ . The  $\tilde{\omega}$ -s are linear combinations of body-fixed angular velocities  $\omega$ -s, see (12) below. The expression for static energy can be obtained from this one by substitution  $\vec{s}_{ik} = 2[\vec{L}_i \vec{L}_k]$ . Note that in [1] few terms in the expression  $E_{Sk}$  proportional to  $\sqrt{3}/2$  are missed by misprint. The explicit expressions for  $L_i$  depend on the ansatz for  $SU(3)$ -matrix  $U$ .

The moments of inertia of the system (8 diagonal and 28 off-diagonal) can be calculated from expressions (10), (11) using the connection between  $\tilde{\omega}$ -s and body-fixed angular velocities of rotation in  $SU(3)$ -configuration space  $\omega$ -s:

$$\tilde{\omega}_i = (R_{ik}(V^\dagger) - R_{ik}(T))\omega_k \quad (12)$$

Real orthogonal matrices  $R_{ik}$  are defined as

$$R_{ik}(V) = \frac{1}{2} Tr \lambda_i V \lambda_k V^\dagger \quad (13)$$

and  $R_{ik}(V^\dagger) = R_{ki}(V)$ .  $U_0 = VT$ ,  $V = U(u, s) \exp(ia\lambda_2)$ ,  $T = \exp(ib\lambda_3)U(d, s)$ . The expressions for the moments of inertia are too bulky to be reproduced here, and we calculated them numerically using (10), (11) and (12), (13) avoiding the explicit analytical expressions for them. For the strange skyrmion molecule we obtained that there are 4 different diagonal inertia,  $\Theta_1 = \Theta_2 = \Theta_N$ ,  $\Theta_3$  which is about 0.3 smaller than  $\Theta_1$ ,  $\Theta_4 = \Theta_5 = \Theta_6 = \Theta_7 = \Theta_S$  and  $\Theta_8$  which is a bit greater than  $\Theta_4$  [4]. In view of these symmetry relations the dipole-type biskyrmion can be quantized like the configuration possessing axial symmetry, at least for the lowest possible value of angular momentum,  $J = 0$ .

To obtain quantization conditions for skyrmions with arbitrary strangeness content the Wess-Zumino term in the action should be investigated [4]. The quantization condition for  $SU(2)$  skyrmions located incidently in  $(u, d)$  subgroup and quantized with  $SU(3)$  collective coordinates was obtained previously in [6] and has the form

$$Y_R = N_c B / 3 \quad (14)$$

where  $Y_R$  is the so called right hypercharge characterizing the  $SU(3)$  irrep under consideration,  $N_c$  is the number of colors in the underlying QCD,  $N_c = 3$  usually,  $B$

is the baryon number of the system. This quantization condition can be generalized to

$$Y_R^{min} = \frac{2}{\sqrt{3}} dL^{WZ}/d\omega_8 = N_c B(1 - 3C_S)/3 \quad (15)$$

where  $C_S$  is the scalar strangeness content of the soliton [4].

The formula (15) can be obtained from the Wess-Zumino term written in the elegant form by Witten [8]. It is valid exactly for any soliton obtained from  $(u, d)$ - $SU(2)$  - solitons by means of rotation into "strange" direction. In this case  $C_S = \frac{1}{2} \sin^2 \nu$ ,  $\nu$  being the angle of rotation. (15) is valid also for  $SO(3)$  solitons considered in [7]. For these solitons  $SC = \frac{1}{3}$  and  $Y_R = 0$ . (15) was checked numerically for the dipole-type configuration with  $C_S = 0.5$  found recently in [1]. The Wess-Zumino term which defines the quantum numbers of the configuration also can be expressed through the currents  $L_{k,i}$ . In general case there are 8 different Wess-Zumino functions according to the relation

$$L^{WZ} = L_i^{WZ} \omega_i = \frac{N_c}{24\pi^2} \int (R_{ik}(U_0) + \delta_{ik}) W_k \omega_i d^3x \quad (16)$$

The summation over coinciding indices is assumed here,  $i, k = 1, \dots, 8$ .

The real orthogonal matrix  $R_{ik} = \frac{1}{2} Tr \lambda_i U_0 \lambda_k U_0^+$ . The most important for us is function  $W_8$

$$W_8 = -\sqrt{3}(L_1 L_2 L_3) + (L_8 L_4 L_5) + (L_8 L_6 L_7) \quad (17)$$

$(L_1 L_2 L_3)$  is the mixed product of the vectors  $\vec{L}_1, \vec{L}_2, \vec{L}_3$ . To calculate the functions  $W_k$  as well as expressions (10), (11) above we used the expressions for the Cartan-Maurer currents of the dipole-type configuration defined as  $TU_0^\dagger d_i U_0 T^\dagger = iL_{k,i} \lambda_k$ . We obtained [1]:

$$\begin{aligned} L_{1i} &= s_a c_a l_{3i}, & L_{2i} &= d_i a, \\ L_{3i} &= (c_{2a} l_{3i} - r_{3i})/2 + d_i b, & L_{4i} &= c_a l_{1i}, \\ L_{5i} &= c_a l_{2i}, & L_{6i} &= s_a l_{1i} + r_{1i}(b), \\ L_{7i} &= s_a l_{2i} + r_{2i}(b), & L_{8i} &= \sqrt{3}(l_{3i} + r_{3i})/2. \end{aligned} \quad (18)$$

Here  $r_1(b) = r_1 c_b - r_2 s_b$ ,  $r_2(b) = r_1 c_b + r_2 s_b$ ,  $l_{ki}$  and  $r_{k,i}$  are left and right  $SU(2)$  C-M currents,  $k, i = 1, 2, 3$ .

In this case the following relation takes place approximately:

$$Y_R^{min} = \frac{2}{\sqrt{3}} dL^{WZ}/d\omega_8 = -(B_L + B_R)/2 \quad (19)$$

where  $B_L$  and  $B_R$  are the baryon numbers located in left  $(u, s)$  and right  $(d, s)$   $SU(2)$  subgroups of  $SU(3)$ , or vice versa. In the case we investigate  $B_L = B_R = 1$ . We obtained the relation (19) numerically [4], but we guess that it can be obtained also analytically.

After standard quantization procedure [6],[7] the simplified expression for the rotation energy of the dipole solitons is

$$E_{rot} = \frac{C_2(SU_3) - 3Y_R^2/4}{2\Theta_S} + \frac{N(N+1)}{2} \left( \frac{1}{\Theta_N} - \frac{1}{\Theta_S} \right) + \frac{3(Y_R - Y_R^{min})^2}{8\Theta_8} \quad (20)$$

$C_2(SU_3) = (p^2 + q^2 + pq)/3 + p + q$  is the second order Casimir operator of  $SU(3)$  group depending on the numbers of upper and lower indices  $p$  and  $q$  in the tensor describing the  $SU(3)$  irrep  $(p, q)$ .  $N$  is the right isospin equal to the isospin of the isomultiplet with  $Y = Y_R$ . The difference between  $\Theta_N$  and  $\Theta_3$  is neglected here for the estimate as well as few interference moments of inertia different from zero, e.g.  $\Theta_{46}$  and  $\Theta_{57}$  which are at least one order of magnitude smaller than diagonal inertia. The angular momentum of configurations we discuss has the lowest possible value  $J = 0$ .

It is clear that for  $\Theta_8 = 0$  it should be  $Y_R = Y_R^{min}$ , otherwise this quantum correction will be infinitely large. With  $Y_R^{min}$  given by (14) this is just the Guadagnini's quantization condition [6].

For  $Y_R = Y_R^{min} = -1$  the following  $SU(3)$  irreps for dibaryons are allowed: octet  $(p, q) = (1, 1)$ , decuplet  $(3, 0)$  and antidecuplet  $(0, 3)$ . The corresponding energies including static mass of the soliton and  $E_{rot}$  are equal to 4.44, 5.0 and 5.5 *Gev* which should be compared with central values of masses of baryons octet and decuplet 2.64 and 3.05 *Gev*. The  $SU(3)$  irreps with  $Y_R$  different from  $Y_R^{min}$  are also allowed. Many of these states should be bound relative to the strong interactions, see also discussion of the Casimir energy below.

The mass splittings within  $SU(3)$  multiplets of dibaryons obtained from strange skyrmion molecule are defined by  $FSB$  terms in the lagrangian, as usually, and are within 150 – 200 *Mev*.

Note that for the case of quantization of  $SU(2)$  bound skyrmion in  $SU(3)$  collective coordinates space the following multiplets appeared with  $Y_R = B$ , for  $B = 2$ : antidecuplet  $(0, 3)$ , 27-plet  $(2, 2)$ , 35-plet  $(4, 1)$ , 28-plet  $(6, 0)$ . All these multiplets have nonstrange components, and the ratio of strangeness to  $B$ -number changed down to  $-3$  in the 28-plet [9].

5. The investigations we performed are not complete because we cannot make a statement that all possible  $B = 2$  configurations in  $SU(3)$  flavor space have been investigated.

However, we established connection between  $SO(3)$  hedgehog and  $SU(2)$ -torus and found that both are local minima in  $SU(3)$  configuration space. New local minimum with large strangeness content found recently is investigated also in the flavor symmetry broken case.

The quantization of this new configuration is performed. The known previously quantization condition is generalized for the states with strangeness content different from zero. The lowest  $SU(3)$  multiplets which appear in this case do not contain nonstrange components, they are in some sense more "strange" than multiplets obtained with quantization condition [6].

The uncertainty in the values of binding energies of states obtained by means of quantization of the strange skyrmion molecule [1] is the smallest one in comparison with other states [7], [9] due to cancellations of the poor known Casimir energies which control the absolute values of masses, according to present understanding [10],

[11]. Indeed, the molecule consists of two slightly deformed unit skyrmions, therefore the Casimir energy of the molecule should be close to approximately twice of that of unit hedgehog and should not make big influence on the binding energies. The results we obtained [4] are in general qualitative agreement with the results obtained recently in [12] where the interaction potential of two hyperons was estimated at relatively large distances. More detailed calculations are being in progress now.

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### Figure captions

Fig.1 Equal mass density lines for the dipole-type configuration with  $SC \approx 0.48$ , binding energy  $(\simeq 70 \pm 5)\text{MeV}$ ,  $d_0 \simeq 0.75\text{Fm}$ , plane  $x = 0$ ,  $F_\pi = 186\text{MeV}$ ,  $e = 4.12$

Fig.2 The function  $a$  defining the relative local orientation of deformed  $(u, s)$  and  $(d, s)$  hedgehogs in isospace. Plane  $z = 0$ , a)  $d_0 = 0.75\text{Fm}$ ,  $M = 3.90\text{Gev}$  b)  $d_0 = 0.02\text{Fm}$ ,  $M = 4.02\text{Gev}$ .

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